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THEORETICAL DETERMINATION OF LIFETIME OF COMPRESSED PLATES AT ELEVATED TEMPERATURES

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SUMMARY

A method for the theoretical determination of the lifetime of compressed plates at elevated temperatures is presented. In this approach, linearized equations are used throughout with the assumption that the plate material is a standard linear solid. The critical time (lifetime) is determined by reducing the time-dependent behavior to the time-independent response of purely elastic buckling.

Theoretically predicted lifetimes of 2024-T3 (formerly 24S-T3) aluminum-alloy plates at 450° F are compared with experimental values obtained in previous work.

INTRODUCTION

With the increase of flight speeds, it becomes more and more important to investigate extensively the behavior of compressed load-carrying structural components such as columns and plates. At high speeds one of the effects of aerodynamic heating consists in changing the properties of many structural materials from elastic to viscoelastic. Accordingly, numerous papers dealing with the problem of such time-dependent behavior, particularly creep bending and buckling, appeared during the last decade. A review of these and related questions was presented recently by Hoff (ref. 1) and Shanley (ref. 2).

As has been repeatedly observed (refs. 1 and 2), one of the essential features of such time-dependent response is the existence of a critical time at which the structure fails under constant compressive loads. A variety of viewpoints may be adopted to determine analytically this critical time. For example, expressions for the deflection of a compressed element can be found, and the critical time can be defined as the time at which this deflection becomes infinite. Since in this case the use of a nonlinear stress-strain relation (or creep law) is required and since the experimental creep curve for metals is nonlinear, the use of nonlinear stress-strain relations possibly may be required in order to determine analytically any critical time.

Another approach could consist in assuming an initial imperfection that would increase in time. The critical time (or lifetime) could then be defined, still with a linear stress-strain relation, as the time at which a certain allowable quantity (such as stress, strain, or deflection) is reached.

Still another point of view is adopted in the present study. It is assumed that the plate is perfectly flat and that the constant compressive stresses will cause time-dependent decrease of Young's (or shear) modulus, which in turn is proportional to the flexural rigidity. This decrease is determined by developing the plate equations on the basis of stress-strain relations derived from spring-dashpot models. The critical time is defined as the instant at which the flexural rigidity reaches a value that would make a corresponding elastic plate unstable under the given loading.

This approach resembles somewhat the use of the secant modulus in the analysis of plastic buckling and is difficult to justify on purely theoretical grounds. The main advantage is that it conceptually reduces the time-dependent behavior to a purely elastic, time-independent response without introducing any new buckling or failure criteria. Therefore, a high degree of idealization is embodied, and the physical reasonableness can be tested only by comparison with experimental results; but its merits for the aircraft structural engineer are obvious.

Another feature of the present approach, which contrasts it to the other methods, consists in the fact that no initial imperfections have to be assumed. This is of some significance, since experimental results (ref. 3) show that initial deviations from flatness do not influence appreciably the lifetime of compressed plates.

The basic equations governing the behavior of viscoelastic plates in compression are derived and discussed in the first section of this report. The second section establishes the expressions relating the applied compressive force to the lifetime of the plate.

In the third and last section a quantitative comparison of the theoretical predictions and experimental results is carried out. The material constants necessary for such a comparison were extracted from a completely different source (ref. 4) than that giving the experimental results of the lifetime of plates (ref. 3).

Because of the scarcity of information on material properties and lifetimes and because of the large scatter to be expected in such experiments, this first quantitative comparison, which gives the correct order of magnitude and the correct dependence on the parameters, may be considered as promising.

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SYMBOLS

a,b	edge lengths of plate
D_e	elastic flexural rigidity of plate
D_h,D_l	upper and lower flexural rigidity of plate, respectively
E	Young's modulus
F	uniform edge thrust per unit length of plate
F_e	elastic critical load
F_h,F_l	upper and lower critical load, respectively
G	shear modulus
h	plate thickness
K	bulk modulus
k	numerical factor, buckling value
M_x,M_y,M_{xy}	plate moments defined by equation (2)
P,Q	linear differential operators
t	time
t_{cr}	lifetime (critical time)
w	deflection of plate
x,y,z	rectangular Cartesian coordinates
ν	Poisson's ratio
ε_d	strain deviator
$\varepsilon_x,\varepsilon_y,\varepsilon_z,\gamma_{xy}$	strain components
η	coefficient of viscosity introduced in figure 2(b)

μ_1, μ_2	moduli of rigidity introduced in figure 2(b)
σ_d	stress deviator
$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}$	stress components
τ_ε	time of relaxation of stress under constant strain
τ_σ	time of relaxation of strain under constant stress

Subscripts:

cr	critical
e	elastic plate
h	upper
l	lower
r	relaxed (modulus)
u	unrelaxed (modulus)

BASIC EQUATIONS

A flat, rectangular plate with edge lengths a and b and thickness h is referred to the system of coordinates shown in figure 1 and is loaded along the edges $x = 0,a$ by a uniformly distributed (compressive) force per unit length F. All four edges are simply supported.

The stress equation of equilibrium in the transverse direction z is (e.g., see ref. 5)

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\, \partial y} + \frac{\partial^2 M_y}{\partial y^2} = F\frac{\partial^2 w}{\partial x^2} \tag{1}$$

where the moments are defined in the usual way:

$$M_x = \int_{-h/2}^{h/2} \sigma_x z\, dz; \quad M_y = \int_{-h/2}^{h/2} \sigma_y z\, dz; \quad M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} z\, dz \tag{2}$$

and w is the deflection. In the present linearized analysis, the force F is not related to w. To express equilibrium (eq. (1)) in terms of one single unknown (deflection w), the stress-strain relations of the material and the geometrical strain-displacement relations have to be specified.

In establishing the relations between the moments M_x, M_y, M_{xy} and the deflection w, assume that the plate material is linearly viscoelastic. In the absence of volume viscosity, the stress-strain relations for such a material may be written down in the form (see ref. 6):

$$\left.\begin{aligned} P\sigma_d &= 2Q\varepsilon_d \\ \sigma_x + \sigma_y + \sigma_z &= \frac{E}{1 - 2\nu}\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \end{aligned}\right\} \tag{3}$$

where σ_d is the stress deviator, ε_d is the strain deviator, σ_x, σ_y, and σ_z are the normal components of stress, and ε_x, ε_y, and ε_z are the elongations. The quantity $E/3(1 - 2\nu)$ is the bulk modulus K, and P and Q are the linear differential operators in time:

$$\left.\begin{aligned} P &= a_0 + a_1 \frac{\partial}{\partial t} + \ldots + a_m \frac{\partial^m}{\partial t^m} \\ Q &= b_0 + b_1 \frac{\partial}{\partial t} + \ldots + b_n \frac{\partial^n}{\partial t^n} \end{aligned}\right\} \tag{4}$$

The stress-strain relations (eq. (3)) involving normal stresses and strains may be written out in scalar form as

$$\left.\begin{aligned} P\sigma_x &= \left[\frac{E}{1 - 2\nu}\frac{P}{3} - \frac{2}{3}Q\right] e + 2Q\varepsilon_x \\ P\sigma_y &= \left[\frac{E}{1 - 2\nu}\frac{P}{3} - \frac{2}{3}Q\right] e + 2Q\varepsilon_y \\ P\sigma_z &= \left[\frac{E}{1 - 2\nu}\frac{P}{3} - \frac{2}{3}Q\right] e + 2Q\varepsilon_z \end{aligned}\right\} \tag{5}$$

By eliminating ε_z in the preceding set and neglecting σ_z,

$$\left.\begin{aligned} P\left(\frac{E}{1-2\nu}\frac{P}{3}+\frac{4}{3}Q\right)\sigma_x &= \left(\frac{2E}{1-2\nu}\frac{P}{3}+\frac{2}{3}Q\right)2Q\varepsilon_x + \left(\frac{E}{1-2\nu}\frac{P}{3}-\frac{2}{3}Q\right)2Q\varepsilon_y \\ P\left(\frac{E}{1-2\nu}\frac{P}{3}+\frac{4}{3}Q\right)\sigma_y &= \left(\frac{2E}{1-2\nu}\frac{P}{3}+\frac{2}{3}Q\right)2Q\varepsilon_y + \left(\frac{E}{1-2\nu}\frac{P}{3}-\frac{2}{3}Q\right)2Q\varepsilon_x \end{aligned}\right\} \tag{6}$$

The procedure is analogous to the treatment of the elongation ε_z and the stress component σ_z in case of elastic plate materials (e.g., see ref. 7).

By assuming, further, the stress-displacement relations to be those of the classical linear plate theory, namely

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2};\ \varepsilon_y = -z\frac{\partial^2 w}{\partial y^2};\ \gamma_{xy} = -z\frac{\partial^2 w}{\partial x\,\partial y} \tag{7}$$

and by noting that in shear

$$P\tau_{xy} = Q\gamma_{xy}$$

the moment-deflection relations can be found by combining equations (2), (3), and (7):

$$\left.\begin{aligned} P\left(\frac{E}{1-2\nu}\frac{P}{3}+\frac{4}{3}Q\right)M_x &= -\frac{h^3}{12}\left(\frac{2E}{1-2\nu}\frac{P}{3}+\frac{2}{3}Q\right)2Q\frac{\partial^2 w}{\partial x^2} - \frac{h^3}{12}\left(\frac{E}{1-2\nu}\frac{P}{3}-\frac{2}{3}Q\right)2Q\frac{\partial^2 w}{\partial y^2} \\ P\left(\frac{E}{1-2\nu}\frac{P}{3}+\frac{4}{3}Q\right)M_y &= -\frac{h^3}{12}\left(\frac{2E}{1-2\nu}+\frac{2}{3}Q\right)2Q\frac{\partial^2 w}{\partial y^2} - \frac{h^3}{12}\left(\frac{E}{1-2\nu}\frac{P}{3}-\frac{2}{3}Q\right)2Q\frac{\partial^2 w}{\partial x^2} \\ P\left(\frac{E}{1-2\nu}\frac{P}{3}+\frac{4}{3}Q\right)M_{xy} &= -\frac{2h^3}{12}\left(\frac{E}{1-2\nu}\frac{P}{3}+\frac{4}{3}Q\right)Q\frac{\partial^2 w}{\partial x\,\partial y} \end{aligned}\right\} \tag{8}$$

then the equilibrium condition, equation (1) in terms of w only, is obtained by substituting relations (8) into equation (1):

$$\left(\frac{E}{1-2\nu}\frac{P}{3}+\frac{Q}{3}\right)Qh^3\nabla^4 w + 3P\left(\frac{E}{1-2\nu}\frac{P}{3}+\frac{4}{3}Q\right)F\frac{\partial^2 w}{\partial x^2} = 0 \tag{9}$$

To treat any particular initial-value problem, the specific form of the operators P and Q has to be prescribed. In case of the so-called four-parameter model (fig. 2(a)), which was discussed and applied, for example, by Glauz and Lee (ref. 8), these operators are of the form

$$\left.\begin{aligned} P &= \frac{\bar{\mu}_1}{\eta_1} + \left(1 + \frac{\bar{\mu}_1}{\bar{\mu}_2} + \frac{\eta}{\eta_2}\right)\frac{\partial}{\partial t} + \frac{\eta_1}{\bar{\mu}_2}\frac{\partial^2}{\partial t^2} \\ Q &= \bar{\mu}_1 \frac{\partial}{\partial t} + \eta_1 \frac{\partial^2}{\partial t^2} \end{aligned}\right\} \tag{10}$$

where $\bar{\mu}_1$, η_1, $\bar{\mu}_2$, and η_2 are the four constants of the model.

Since in the case of a four-parameter model the operators P and Q are of second order, considerable complications are introduced in solving the equilibrium equation (9) together with the appropriate initial conditions. Since, in the process considered presently, no unloading takes place, the omission of the element representing unrecoverable flow (η_2 in fig. 2(a)) appeared to be justified. The model was thus reduced to a three-parameter one, as shown in figure 2(b). This model was discussed extensively by Zener (ref. 9) and is referred to as the one characterizing a standard linear solid.

The operators P and Q are now of first order:

$$\left.\begin{aligned} P &= 1 + \frac{\eta}{\mu_1}\frac{\partial}{\partial t} \\ Q &= \mu_2\left(1 + \frac{\eta}{\mu_1} + \frac{\eta}{\mu_2}\frac{\partial}{\partial t}\right) \end{aligned}\right\} \tag{11}$$

Several other quantities, defined in terms of the model constants μ_1, μ_2, and η may be introduced for convenience and better physical insight. In particular, it is useful to know the time-dependence of E and ν in terms of the shear modulus G. The quantity $\tau_\varepsilon = \eta/\mu_1$ is the time of relaxation of stress under constant strain, $\tau_\sigma = \eta/\mu_1 + \eta/\mu_2$ is the time of relaxation under constant stress, $G_r = \mu_2$ is the relaxed shear modulus, and $G_u = \mu_1 + \mu_2$ is the unrelaxed shear modulus. It follows that

$$\frac{G_u}{G_r} = \frac{\tau_\sigma}{\tau_\varepsilon} \tag{12}$$

By introducing E_u as the unrelaxed Young's modulus, E_r as the relaxed Young's modulus, K_u as the unrelaxed bulk modulus, and K_r as the relaxed bulk modulus, the absence of volume viscosity requires that

$$K_u = K_r = K$$

But

$$K = \frac{1}{3}\,\frac{E(t)\;G(t)}{3G(t) - E(t)} = \frac{1}{3}\,\frac{E_r\;G_r}{3G_r - E_r} = \frac{1}{3}\,\frac{E_u\;G_u}{3G_u - E_u} \tag{13}$$

Since (ref. 9)

$$E_u = 2(1 + \nu)\;G_u \tag{14}$$

then

$$\frac{3}{E_r} - \frac{1}{G_r} = \frac{1 - 2\nu}{E_u} \tag{15}$$

Evidently

$$E_r \neq 2(1 + \nu)\;G_r$$

since, from equation (13),

$$E(t) = \frac{3E_u\;G(t)}{(1 - 2\nu)\;G(t) + E_u} \tag{16}$$

Some authors (refs. 10 and 11) prefer to use the form

$$E(t) = 2\;[1 + \nu(t)]\;G_u \tag{17}$$

By comparison with equation (16), $\nu(t)$ must be

$$(t) = \frac{(2 + 5\nu)\;G(t) - E_u}{(1 - 2\nu)\;G(t) + E_u} \tag{18}$$

It may be noted that, while $\nu(0) = \nu$, $\nu(\infty) \neq 1/2$ except for a Maxwell model ($\mu_2 = 0$). Further, in the expression

$$\sigma_x + \sigma_y + \sigma_z = 3K(\varepsilon_x + \varepsilon_y + \varepsilon_z) \tag{19}$$

the value $3K$ may be conveniently replaced by $\frac{2(1 + \nu)G_u}{(1 - 2\nu)}$, which is a constant. The quantity ν has here its initial value.

Even though for a three-parameter model the operators P and Q are of first order, equation (9) on the deflection w will be of second order.

An attempt could be made to lower this order while the same basic model is still being considered for the stress-strain relations. This, in fact, can be done by extending the elastic stress-strain relations for a plate directly to the viscoelastic ones without ever considering, as was done before, the three-dimensional stress-strain relations. The physical significance of this latter procedure, as contrasted to the previous one, will be discussed at the end of the next section.

In the present case of a three-parameter model, therefore,

$$\left.\begin{aligned}\left(1 + \tau_\varepsilon \frac{\partial}{\partial t}\right) \sigma_x &= \frac{2G_r}{1 - \nu}\left(1 + \tau_\sigma \frac{\partial}{\partial t}\right)(\varepsilon_x + \nu\varepsilon_y)\\ \left(1 + \tau_\varepsilon \frac{\partial}{\partial t}\right) \sigma_y &= \frac{2G_r}{1 - \nu}\left(1 + \tau_\sigma \frac{\partial}{\partial t}\right)(\varepsilon_y + \nu\varepsilon_x)\\ \left(1 + \tau_\varepsilon \frac{\partial}{\partial t}\right) \tau_{xy} &= G_r\left(1 + \tau_\sigma \frac{\partial}{\partial t}\right) \gamma_{xy}\end{aligned}\right\} \tag{20}$$

This leads to the equation

$$\frac{h^3}{12}\frac{2G_r}{1 - \nu}\left(1 + \tau_\sigma \frac{\partial}{\partial t}\right) \nabla^4 w + \left(1 + \tau_\varepsilon \frac{\partial}{\partial t}\right) F \frac{\partial^2 w}{\partial x^2} = 0 \tag{21}$$

or

$$\frac{h^3}{12}\frac{2Q}{1 - \nu} \nabla^4 w + PF \frac{\partial^2 w}{\partial x^2} = 0 \tag{22}$$

which replaces equation (9) and is of the first order in time.

CONCEPT OF ELASTIC BUCKLING APPLIED TO VISCOELASTIC PLATES

The external force F is assumed to act in the form of a step function and keeps its constant value after the beginning of loading at a time $t = 0$.

One of the features of the linearized elastic stability analysis is that no relation is sought between the force F and the deflection w (ref. 12 contains a careful and detailed analysis of the stability phenomenon).

In the case of an elastic plate, the critical force F_e is found to be (ref. 12)

$$F_e = \frac{D_e \pi^2}{b^2} k \tag{23}$$

where D_e is the flexural rigidity of the elastic plate

$$D_e = \frac{G_e h^3}{6(1 - \nu)} \tag{24}$$

and G_e is the shear modulus of the elastic plate; k is a numerical factor, the buckling value, the magnitude of which depends on the ratio a/b.

In the case of a viscoelastic plate characterized by the three-parameter model (fig. 3), the following statement can be made: Since the plate is assumed to be perfectly flat, the constant force F produces only constant compressive stresses. Further, at the instant of loading the viscosity element is as yet ineffective, and the rigidity is governed by the unrelaxed modulus G_u. On the other hand, if the force F is permitted to act for an infinitely long time, the spring μ_2 becomes ineffective and the rigidity is governed by the relaxed modulus G_r.

Thus an upper critical load F_h, under which the plate buckles instantaneously, and a lower critical load F_l, under which the plate buckles after an infinite time, may be defined. Hence, for any load $F > F_h$ the plate buckles instantaneously; for $F < F_l$ the plate never buckles; and for $F_l < F < F_h$ the plate buckles after a finite period of time, which may be referred to as the lifetime or critical time t_{cr}.

The expressions for F_h, F_l, and t_{cr} may be determined readily. To calculate F_h, let $P = 1$ and $Q = G_u$, and obtain from equation (9)

$$F_h = \frac{D_h \pi^2}{b^2} k \tag{25}$$

where

$$D_h = \frac{G_u h^3}{6(1 - \nu)} \tag{26}$$

To find F_d, let $P = 1$ and $Q = G_r$, and obtain similarly

$$F_l = \frac{D_l \pi^2}{b^2} k \tag{27}$$

where

$$D_l = \frac{\left[\frac{2(1 + \nu)G_u}{(1 - 2\nu)G_r} + 1\right] G_r h^3}{3\left[\frac{2(1 + \nu)G_u}{(1 - 2\nu)G_r} + 4\right]} \tag{28}$$

To calculate the critical time t_{cr} associated with a load $F_l < F < F_h$, proceed as follows: Since the (constant) force F produces constant stresses in the perfectly flat plate, the stress-strain relation (eqs. (20)) takes the form

$$\tau = G_r \left(1 + \tau_\sigma \frac{\partial}{\partial t}\right) \gamma \tag{29}$$

During the interval of time $0 < t < \infty$, the shear modulus $G(t)$ will decrease from G_u to G_r. Since, at $t = 0$, $\tau = G_u \gamma$, then from equation (29)

$$\frac{G(t)}{G_u} = \frac{G_r}{G_u} \frac{e^{t/\tau_\sigma}}{e^{t/\tau_\sigma} + \frac{G_r}{G_u} - 1} \tag{30}$$

The load F as a function of critical time is therefore (again with the use of eq. (9)):

$$\frac{F(t_{cr})}{F_h} = \frac{D(t)}{D_h} = \frac{2(1 + \nu) + (1 - 2\nu)\frac{G(t)}{G_u}}{2(1 + \nu) + 4(1 - 2\nu)\frac{G(t)}{G_u}} \, 2(1 - \nu) \frac{G(t)}{G_u} \tag{31}$$

A plot of $G(t)/G_u$ against a dimensionless time t/τ_σ is presented in figure 3. Poisson's ratio was taken as $\nu = 0.3$. One curve is for $G_r/G_u = 0.9$ (low viscosity) and the other for $G_r/G_u = 0.1$ (high viscosity). Figure 4 shows a plot of $F(t_{cr})/F_h$ against t/τ_σ, again for $G_r/G_u = 0.9$ and $G_r/G_u = 0.1$.

With the simpler equation (22), the same relation (30) is obtained for $G(t)/G_u$; but a different one is obtained for $F(t_{cr})/F_h$, namely

$$\frac{F(t)}{F_h} = \frac{D(t)}{D_h} = \frac{G(t)}{G_u} \tag{32}$$

which now replaces equation (31). Thus, with the simpler equation (30), the curve $F(t)/F_h$ is identical to $G(t)/G_u$, whatever the value of G_r/G_u.

It may be of interest to establish the physical meaning of stress-strain relations (20) as contrasted to stress-strain relations (3). A consequence of the former is equation (31), while a consequence of the latter is equation (32). Thus, in order to pass from equation (31) to equation (32),

$$\frac{\left[2(1+\nu) + (1-2\nu)\dfrac{G(t)}{G_u}\right] 2(1-\nu)}{2(1+\nu) + 4(1-2\nu)\dfrac{G(t)}{G_u}} = 1 \tag{33}$$

Introducing into this expression the bulk modulus K gives

$$K = \frac{E}{3(1-2\nu)} = \frac{2(1+\nu)}{3(1-2\nu)} G_u \tag{34}$$

and solving for K gives

$$K = \frac{2(1+\nu)}{3(1-2\nu)} G(t) \tag{35}$$

or

$$\frac{K(t)}{K_u} = \frac{G(t)}{G_u} \tag{36}$$

that is, the bulk modulus changes in proportion to the shear modulus. Thus, while equations (3) express absence of volume viscosity (K = constant), equations (20) describe a material with both volume and shear viscosity.

COMPARISON OF THEORETICAL PREDICTIONS AND EXPERIMENTAL RESULTS

In order to predict a numerical value for the critical time, three material constants corresponding to the three elements of the model must be known. These constants may be obtained from a creep test.

Figure 5 shows an approximate reproduction of a curve from reference 3, which gives the total deformation in percent against time in hours for a 2024-T3 aluminum sheet at 450° F subjected to a compressive constant stress of 26,000 pounds per square inch. From this plot the unrelaxed Young's modulus E_u is readily obtained by dividing the stress by the instantaneous strain:

$$E_u = \frac{26{,}000}{4\times10^{-3}} = 6.50\times10^{6} \text{ psi} \tag{37}$$

If the plot is extrapolated by assuming that the total deformation will reach 1.6 percent for very large times, the relaxed Young's modulus is obtained:

$$E_r = \frac{26{,}000}{1.6\times10^{-2}} \approx 1.60\times10^{6} \text{ psi} \tag{38}$$

When Poisson's ratio ν is 0.3, the unrelaxed shear modulus G_u is calculated to be

$$G_u = \frac{E_u}{2(1+\nu)} = 2.5\times10^{6} \text{ psi} \tag{39}$$

The reciprocal of the relaxed shear modulus G_r, by formula (15), is

$$\frac{1}{G_r} = \frac{3}{E_r} - \frac{(1-2\nu)}{E_u} = 1.8\times10^{-6} \text{ psi}^{-1} \tag{40}$$

Thus,

$$\frac{G_u}{G_r} = 4.5 \tag{41}$$

To find the third constant, use the special form of stress-strain relations (20) for uniaxial constant compression, namely:

$$\sigma = 2G_r(1 + \nu)\ \varepsilon + 2G_r(1 + \nu)\ \tau_\sigma \dot{\varepsilon} \tag{42}$$

and solve for τ_σ:

$$\tau_\sigma = \frac{1}{\dot{\varepsilon}} \left[\frac{\sigma}{2G_r(1 + \nu)} - \varepsilon\right] \tag{43}$$

By taking from the curve of figure 5 several pairs of values of ε and $\dot{\varepsilon}$ and by using the values of σ and G_r just given, an average τ_σ value of 30 hours may be obtained. The stress σ that causes failure can now be determined as a function of the critical time (or lifetime). To find the upper critical stress $(\sigma_{cr})_u = F_h/h$, use formula (25) with $k = 4$, the buckling value for a long, simply supported plate. Substitution of the numerical values yields

$$(\sigma_{cr})_h = 23.5\ \frac{h^2}{b^2} \times 10^6 \text{ psi} \tag{44}$$

The ratio of this critical upper value to the critical value corresponding to a lifetime of, say, τ is on the basis of the preceding theory the same as the ratio of $G(\tau)/G_u$ given by equation (30). Figure 6 shows a plot of the critical stress against lifetime for four thickness-to-width ratios b/h of 20, 30, 45, and 60.

These purely theoretical lifetime curves can be compared with experimental results presented by Mathauser and Deveikis in reference 3, which are also shown in figure 6. Even though the quantitative discrepancy between the predicted and experimental results is in general fairly large, the theory predicts more than merely an order of magnitude and exhibits the proper dependence on time and on thickness-to-width ratio.

Because of the scarcity of information on material properties of aluminum alloys subjected to compression at high temperatures and information on creep lifetime of plates, this first numerical comparison of theoretical and experimental results can be regarded as satisfactory and promising.

Performing the comparison on a wider scale did not appear possible, because only for the 2024-T3 aluminum-alloy plates at 450° F were, to the authors' knowledge, both material constants and creep lifetimes of plates in compression determined experimentally.

CONCLUDING REMARKS

In conclusion, it should be emphasized that the present report contains a highly simplified and idealized analysis, since the influence of metallurgical changes, nonlinearities, time-dependent loads, and so forth, were not taken into account.

However, the fact that creep analysis is bound to be a highly inexact endeavor should be noted. The large amount of scatter to be expected may be seen, for example, in reference 13, where it is shown that varying the temperature $\pm 10^{\circ}$ F every 7 minutes produced creep rates six times as large as those produced in a steady temperature test at 1800° F. Further, a 15-percent stress variation may change a column lifetime by a factor of 10.

Thus it follows that, from a practical point of view, all that can be expected from a creep analysis is merely an order of magnitude. Precisely for this reason, any attempt to refine the approach by including various effects has to be considered with caution.

Columbia University,
New York, N. Y., March 17, 1957

REFERENCES

1. Hoff, N. J.: High Temperature Effects in Aircraft Structures. Appl. Mech. Rev., vol. 8, no. 11, Nov. 1955, pp. 453-456.

2. Shanley, F. R.: Mechanics of Creep and Design for Members Subjected to Buckling. Proc. for Short Course on Mech. of Creep, The Penn. State Univ., 1954, p. 109.

3. Mathauser, Eldon E., and Deveikis, William D.: Investigation of the Compressive Strength and Creep Lifetime of 2024-T3 Aluminum-Alloy Plates at Elevated Temperatures. NACA Rep. 1308, 1957. (Supersedes NACA TN 3552.)

4. Vawter, F. J., Guarnieri, G. J., Yerkovich, L. A., and Derrick, G.: Investigation of the Compressive, Rearing and Shear Creep-Rupture Properties of Aircraft Structural Metals and Joints at Elevated Temperatures, pt. 1. Tech. Rep. 54-270, WADC, June 1954. (Contract AF-33(616)-190.)

5. Timoshenko, S.: Theory of Plates and Shells. McGraw-Hill Book Co., Inc., 1940.

6. Alfrey, T., Jr.: Mechanical Behavior of High Polymers. Interscience Pub., 1948.

7. Mindlin, R. D.: Influence of Rotatory Inertia and Shear on Flexural Motions of Isotropic, Elastic Plates. Jour. Appl. Mech., vol. 18, no. 1, Mar. 1951, pp. 31-38.

8. Glauz, R. D., and Lee, E. H.: Transient Wave Analysis in a Linear Time-Dependent Material. Jour. Appl. Phys., vol. 25, no. 8, Aug. 1954, pp. 947-953.

9. Zener, Clarence: Elasticity and Anelasticity of Metals. The Univ. Chicago Press, 1948.

10. Read, W. T., Jr.: Stress Analysis for Compressible Viscoelastic Materials. Jour. Appl. Phys., vol. 21, no. 7, July 1950, pp. 671-674.

11. Brull, M. A.: A Structural Theory Incorporating the Effect of Time-Dependent Elasticity. Proc. First Midwestern Conf. on Solid Mech., Univ. Ill., 1953, pp. 141-147.

12. Bleich, F.: Buckling Strength of Metal Structures. McGraw-Hill Book Co., Inc., 1952.

13. Mathauser, Eldon E., and Brooks, William A., Jr.: An Investigation of the Creep Lifetime of 75S-T6 Aluminum-Alloy Columns. NACA TN 3204, 1954.

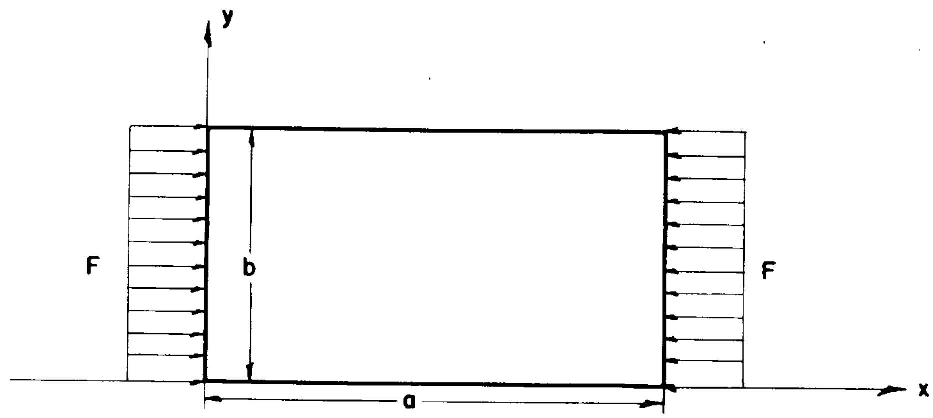

Figure 1. - Compressed plate.

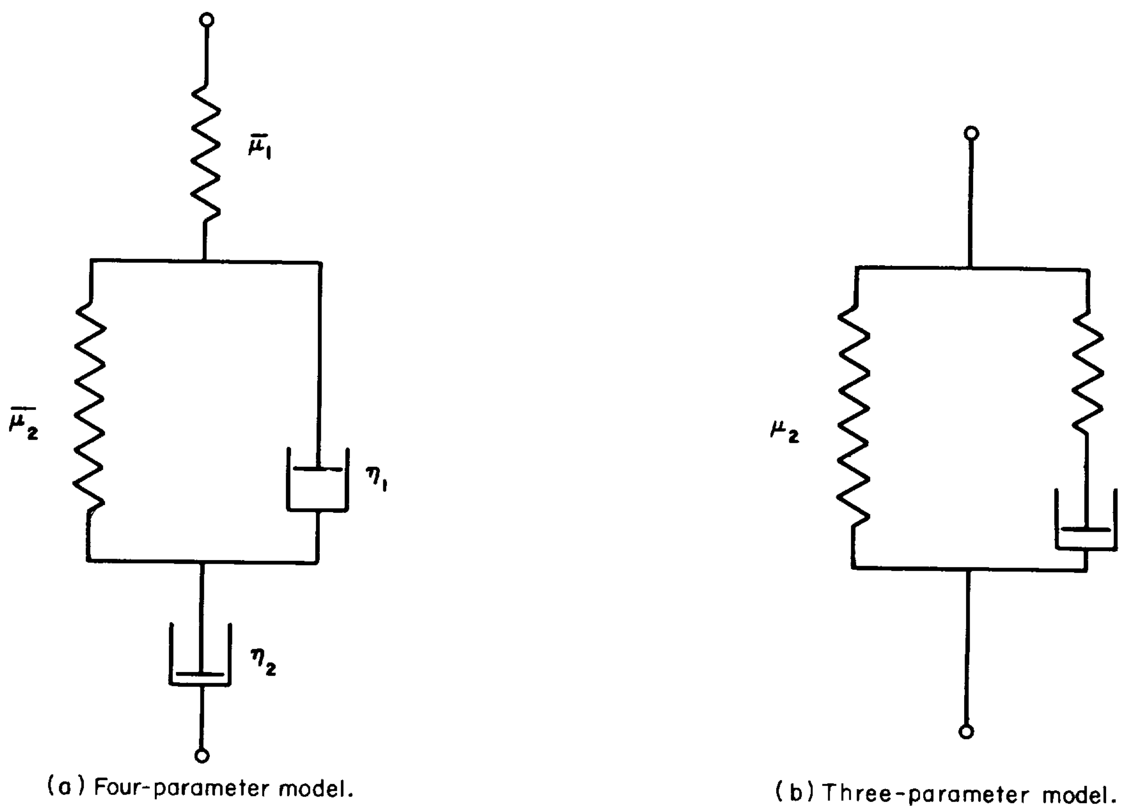

Figure 2. - Viscoelastic models.

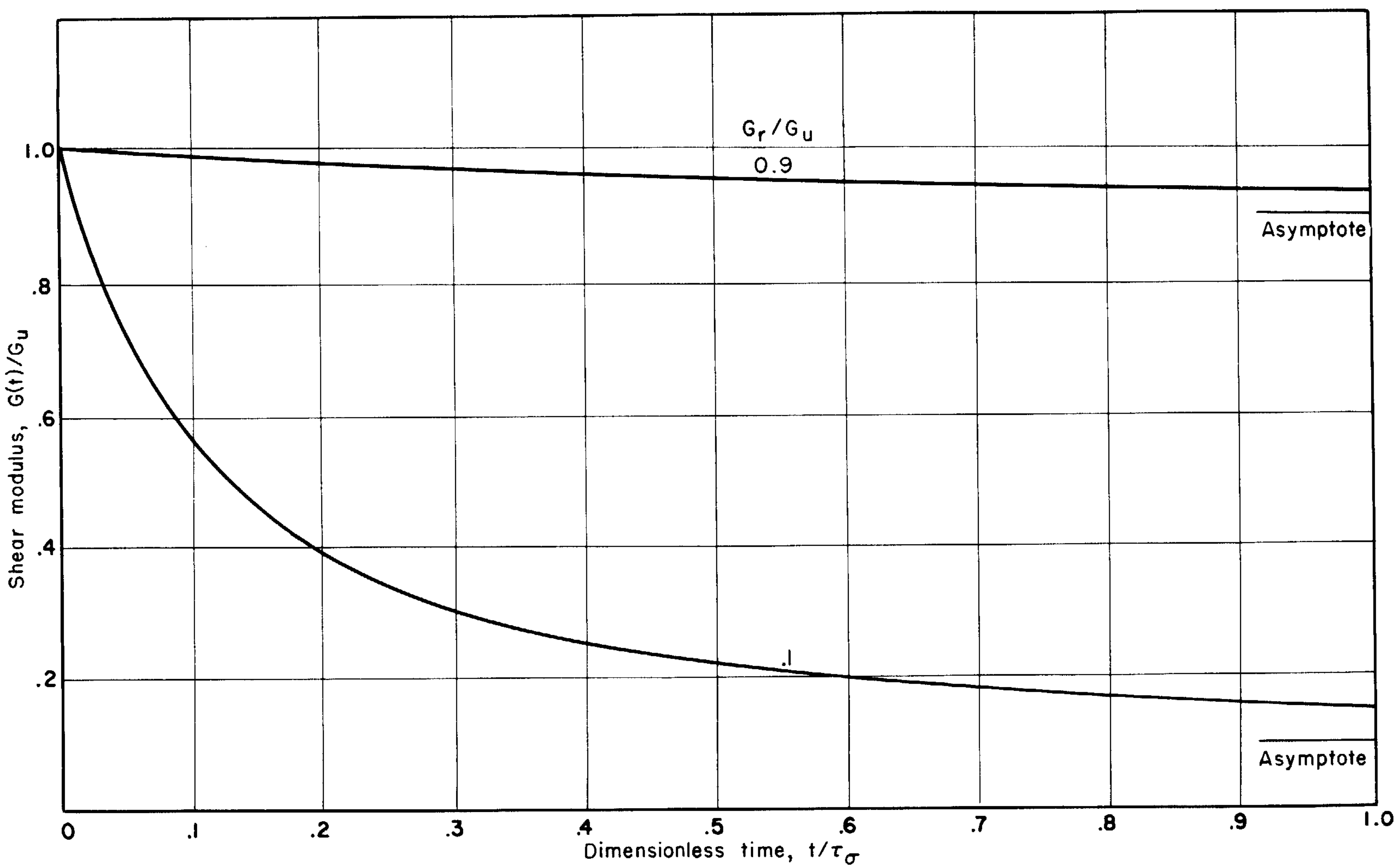

Figure 3. - Time dependence of shear modulus.

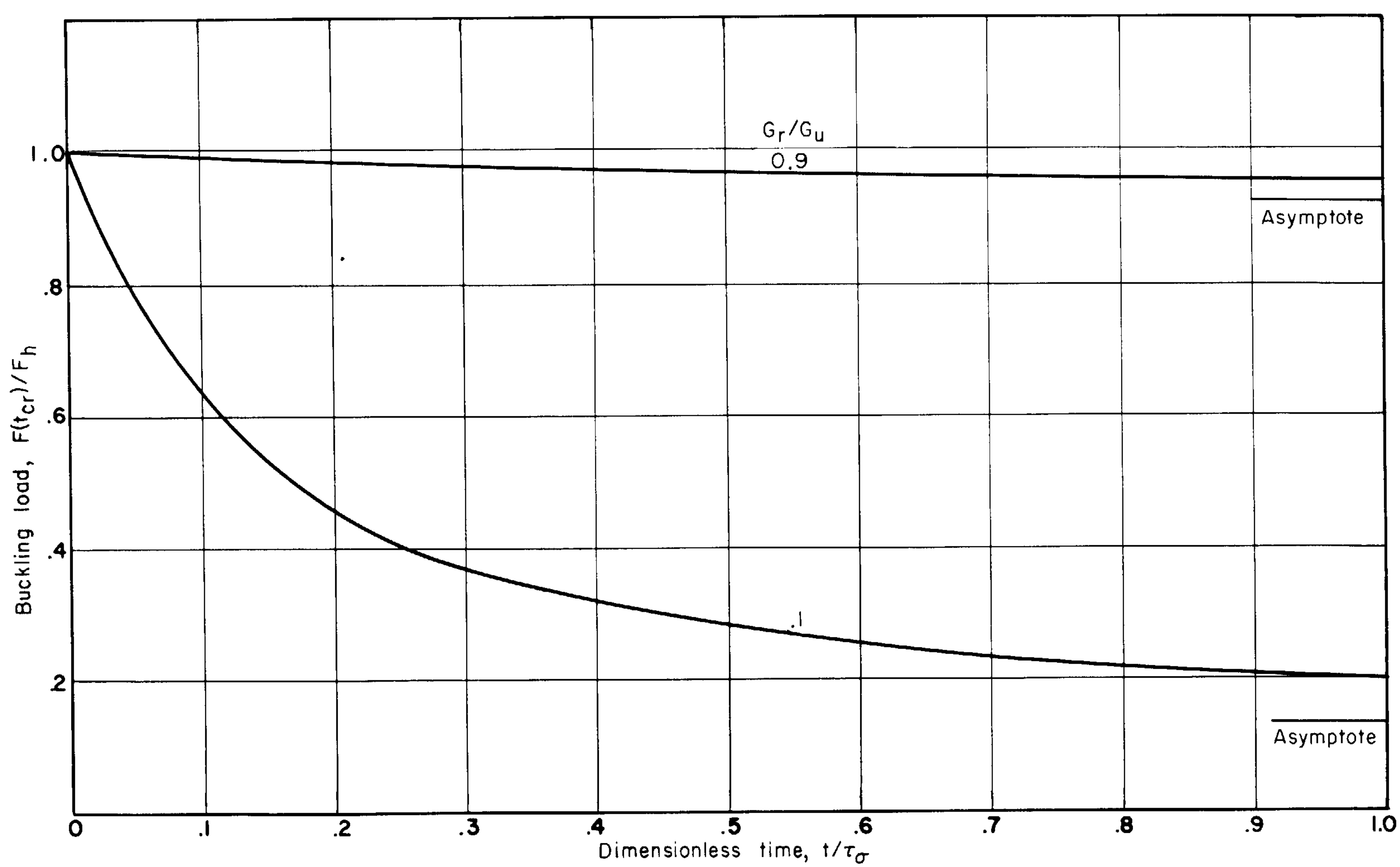

Figure 4. - Buckling load against critical time.

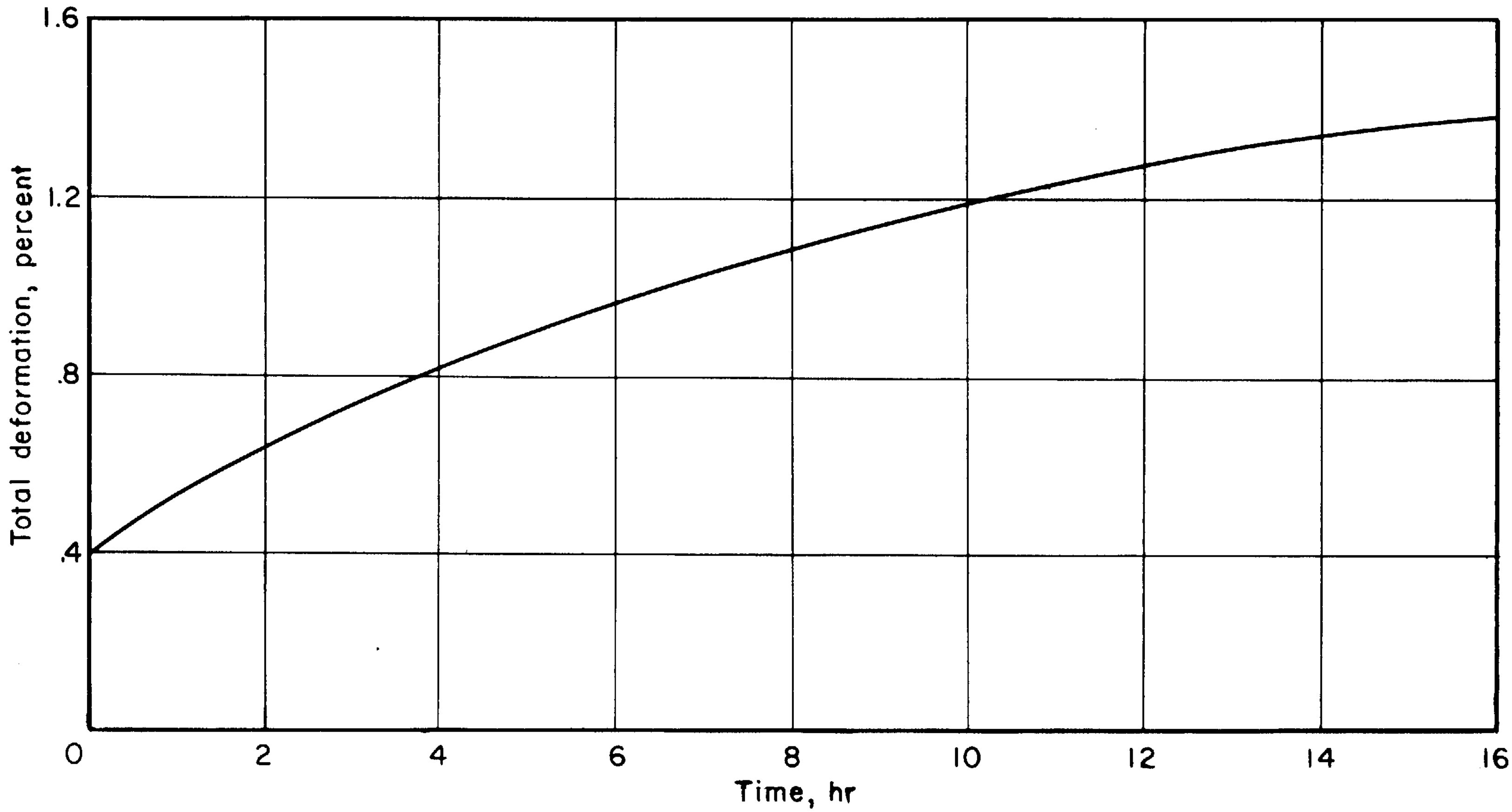

Figure 5. - Compression creep of 2024-T3 aluminum sheet at 450° F and 26,000 pounds per square inch.

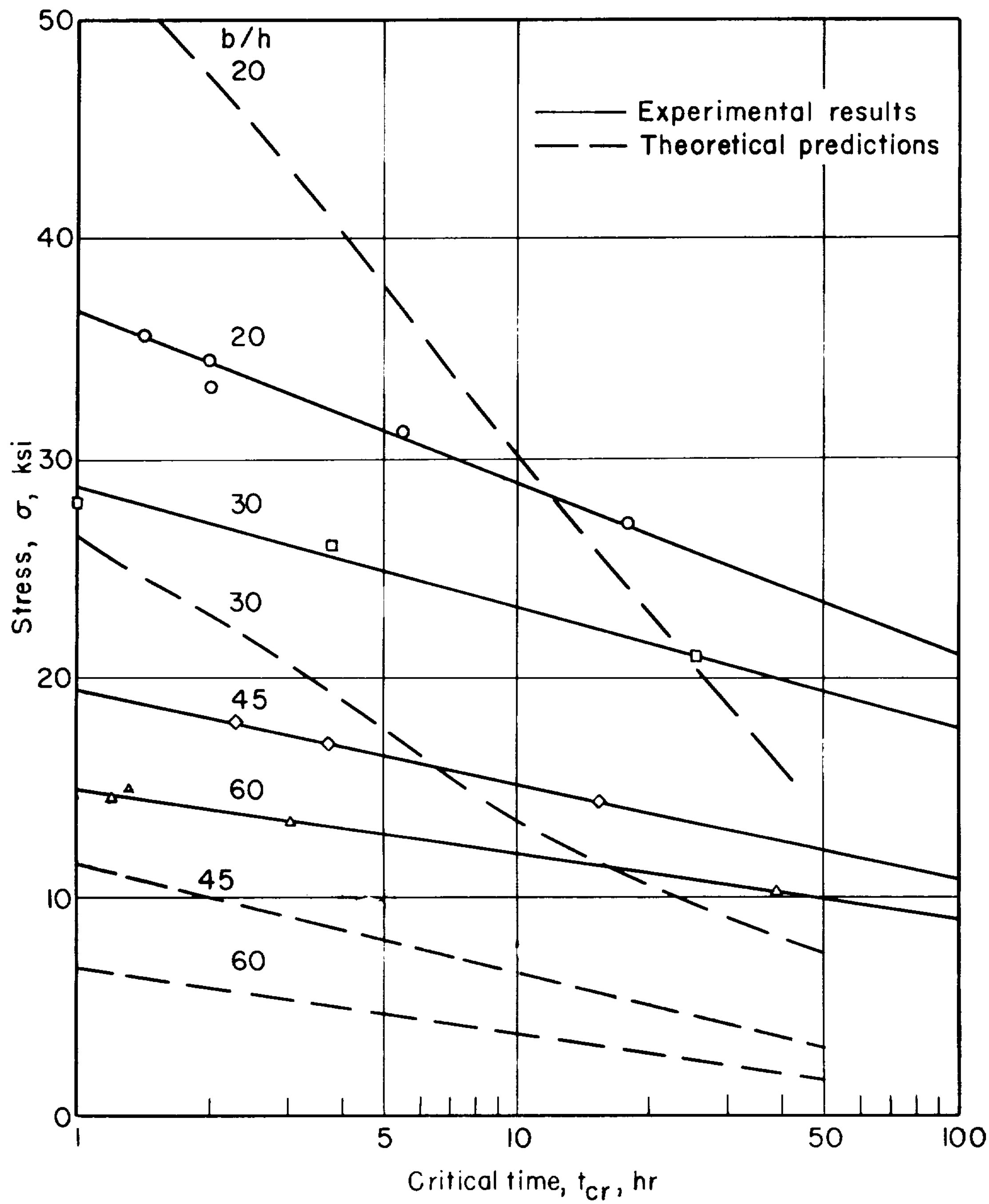

Figure 6. - Comparison of experimental and predicted creep lifetimes for 2024-T3 aluminum-alloy plates. Temperature, 450° F.

NASA MEMO 2-24-59W
National Aeronautics and Space Administration.
THEORETICAL DETERMINATION OF LIFETIME OF COMPRESSED PLATES AT ELEVATED TEMPERATURES. George Herrmann and Hu-Nan Chu, Columbia University. March 1959. 21p. diagrs. (NASA MEMORANDUM 2-24-59W)

The method presented uses linearized equations with the assumption that the plate material is a standard linear solid. The critical time (lifetime) is determined by reducing the time-dependent behavior to the time-independent response of purely elastic buckling. Theoretically predicted lifetimes of 2024-T3 aluminum-alloy plates at 450° F are compared with experimental values obtained in previous work.

Copies obtainable from NASA, Washington

1. Plates, Flat (4.3.3.1)
2. Loads and Stresses, Structural - Compression (4.3.7.2)

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